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Domination of Graph Theory and its Applications

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ABSTRACT: A set of vertices S in graph G dominates G if every vertex in G is either in S or adjacent to a vertex in S. The size of any smallest dominating set is called the domination number of G. Two variant concept that have attracted recent interest are total domination and connected domination. A set of vertices S is a total dominating set if every vertex in the graph is adjacent to a vertex of S and S is a connected dominating set if it is domin and, addition, induces a connected sub graph. The size of any smallest the size of a smaller connected dominating set in G is called the total domination number of G and the size of a smaller connected dominating setis the connected domination number of G.

INTRODUCTION

The theory graph is one of the new field of mathematics. While the history of mathematics graph of Euler's solution of Konisberg bridge problem. Any mathematical object having points and connections between them may be called graphs. Graphs are serve as mathematical modules to analyze Mathematics graph theory is a study of graphs, which are mathematical structure used to model pairwise relation between objects. In 1850, chess freaks in Europe give thought to issue for finding th board with a goal that each one of the blocks are either charged by a queen or in habited by a queen. The "study of the dominating sets in graphs. Also general as the domination of vertices of a graph. As re begin center ported by W. W. Rouse Ball in 1892, chess enthusiasts in the late 1800s studied, the following 3basic types of problem. Graph theory is most accepted applications in various fields in Mathematics and other subjects. The publications of last thirty years show that Graph theory is the fastest growing area among all the subjects in all disciplines. Many real world situations can c d by means of a diagram consisting of a set of points a graph. The purpose of this chapter is to list the terminology and notation that we shall use in this work. Much of the terms used are standard graph theoretic terminology, a introduced later when their turn comes.

DEFINITIONS

- A graph is an ordered triplet G = {v(G), E(G), IG) Where V (G) is a non empty set. E(G) is a set disjoined from V (G) and Io is an incident map that associates with each element of E(G) unordered pair of elements of V (G).V (G) is the vertex set and E(G) is the edge set.

- A graph G = (V (G), E (G)) is a subgraph of G iff $V' \subseteq V$ and $E' \subseteq E$, -+where V (G) is the vertex set and E (G) is the edge set of G Vertices are sometimes called points or nodes and edges are sometimes called lines. Number of vertices n(G)is called the order of the graph G and number of edges m(G) is the size of the graph.

- If Ic(e) = {u, v) then the vertices u and u are called the end vertices of e.

- Vertices u and u are adjacent to each other in G if and only if there is an edge e with u and u as its ends or, two edges e and *f* are said to be adjacent if and only if they have common end vertex.

- The degree of a vertex v in a graph G is the number of edges incident with u and is denoted deger or simply deg v if the graph is clear from the context.

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- ◻ A vertex of degree 0 is referred to as an isolated vertex and the vertex of degree 1 is an end-vertex (or a leaf).The minimum degree of G is the minimum degree among the vertices of G and is denoted by $\delta(G)$.The maximum degree of G is denoted by $\Delta(G)$. So, if G is a graph of order n and u is any vertex of G, then

THE DOMINATION NUMBER ON A GRAPH

Definition. Let G = (V,E) be a graph. A vertex ve Vis said to dominate itself and each of its neighbours, that is a V dominates the vertices in its closed neighbourhood N[V].Therefore v dominates 1 + degv vertices of G.A set S of vertices of G is a dominating set of G if every vertex of G is dominatedby some vertex is S.

That is, a set $S \subseteq V$ of vertices of a graph G = (V,E) is a dominating set if every vertex $v \in V$ is an element of S or adjacent to an element of S. Equivalently, a set S of vertices of G is a dominating set of G if every vertex in V\S is adjacent to some vertex in S.

Definition. A minimum dominating set in a graph G is a dominating set of minimum cardinality.
The cardinality of minimum dominating set is called the domination number of G and is denoted by y(G).The minimum dominating set is called y -set.

Example. Consider Fig, S.- {u, v, w), S2 = {u, U4, U1, U4} are examples of dominating
sets.

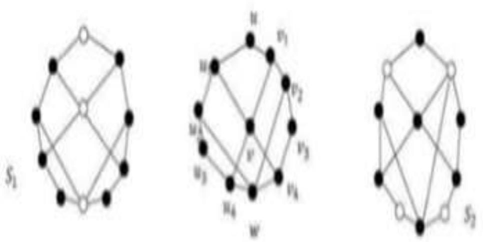

Figure: Examples of dominating sets

Bounds of domination number

Since each isolated vertex in a graph G can only be dominated itself, every dominating set in G must contain its isolated vertices. For graphs without isolated vertices, however there are always 2 disjoint dominating sets.

Theorem

Let G be a graph without isolated vertices. If S is a minimal dominating set of G, then V(G)- S is a dominating set of G.

Proof

Let G be a graph without isolated vertices. Assume that S is a minimal dominating set. We show that V(G)-S is a dominating set of G.

Let vЄ V(G)

If v $\in$ V(G)-S, then V is dominated by itself. Thus we may assume that $v \notin V(G) - S$. We show that V is dominated by some vertex in V (G) - S. Assume, to the contrary, that V is not dominated by any vertex in V (G)-S. Therefore V is not adjacent to any vertex in V (G)-S. Since S is a dominating set in G, each vertex in V (G) is dominated by some vertex in S, different from V. Thus each vertex in V (G) - S is dominated by some vertex in S-V.

On the other hand, G has no isolated vertices and so V is not an isolated vertex of G. Since V is not adjacent to any vertex in V (G)- S. Therefore S -V is a dominating set of G, which contradicts the fact that S is a minimal dominating set of G. For graphs without isolated vertices, there is an upper bound for the domination number of a graph in terms of its order.

TYPES OF DOMINATION

The variations of domination are mainly formed by imposing additional conditionon S, V (G) - S or V (G).We will review some of these variations in the next chap- ter, but our main focus is two type domination, namely, Open domination/Total domination and Independent domination.

Open domination/Total Domination

We have seen that a vertex u dominates a vertex v in a graph if either v = u or v is a neighbour of u. However there are a variations of domination. In this context, we restrict domination so that a vertex u is only permitted to dominatea vertex v if u is a neighbour of u. We refer this type of domination as open domination, although the term total domination is used as well.

Definition : fw Є N (V),then we say that v openly dominates w. That is, a vertex v openly dominates the vertices in its open neighbourhood N(V).

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A set of vertices in a graph G is an open dominating set of G if every vertex of G is adjacent to atleast one vertex of S.

Definition : The minimum cardinality of an open dominating set is the open domination number Yo(G) of G.
An open dominating set of cardinality Yo(G) is a minimum dominating set.

Varieties of Domination
Here we will consider a variety of conditions that can be imposed either on the dominated set V-S, or on the method by which the vertices in V-S are dominated. These include the following.

(1) Multiple domination: In which we insist the each vertex in V - S bedominated by atleast K vertices in S for a fixed positive integer K.

(2)Locating domination: In which we insist that each vertex in V - S hasa unique set of vertices in S which dominate it.

(3)Distance domination: In which we insist that each vertex in V-S bewithin distance K of least one vertex in S for a fixed positive integer K.

(4) Strong domination: In which we insist that each vertex v in V -S bedominated by at least one vertex in S whose degree is greater than or equal to the degree of v. A similar notion weak domination specifies that each vertex v In V-S dominated by at least one vertex in S whose degree is less than or equal to the degree of v.

(5) Global domination: In which we insist that the domination set S also dominates the vertices V-S in the complement of G.

(6) Directed domination: In digraphs in which we insist that for each vertex v in V- S, there is a directed edge from u to v for at least one vertex u in S.

(7) Power domination: A subset $S \subseteq V$ is a power dominating set of G if all vertices of V can be observed recursively by the following rules:
(a) All vertices in N[S] are observed initially.

APPLICATIONS IN GRAPH THEORY

Domination in graphs has applications to several fields. Domination arises in facility location problems, where the number of facilities (eg.. hospitals. tire stations..) is fixed and one attempts to minitmie the distance that a person needs to travel to get to the closest facility. A similar problem occurs when the maxi- mum distance to a facility is fixed and one attempts to minimize the number of facilities necessary so that everyone is serviced.05) Concepts from domination also appear in problem invoking finding sets of representatives, in monitoring communication or electrical network and in land sur-veying
(leg: minimizing the number of places a surveyor must stand in order to take height measurements for an entire region).

College Bus Routing:
Most college in the country provide college buses for transporting children to and from college. Most also operate under certain rules, one of which usually states that no students shall have to walk farther than. say one quarter Kin to a bus pickup point.

Thus, they must construct a route for cacti but that gets within one quarter Km of every student in its assigned area No but ride can take more than some specified number of minutes. and limits on the number of students than a bus can cam' at an one time Let us assume that the college has decided that no students shall have to walk more than two blocks in order to he picked up by a college bus. Construct a route for a college bus that leaves the college. gets within two blocks of every student and returns to the school.

CONCLUSION

Graph Theory is a wide area with more application to real life. Dominations in graph helps the researchers to get more ideas to manage the problem in the real life situation. It has numerous application in modern science and engineering.

An essential part of the motivation in the various applications are based on the varieties of domination. There are more than 75 variation of domination cited in (6) variations are mainly formed by imposing additional condition on S,V (G)-S or V (G).

In this paper, I attempted to categorize domination concepts into some categories. Also domination is an area in graph theory with an extensive research activity. So I conclude this project by introducing some applications of domina- tion in real life situations.

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